$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{2^m\left(2^m+2^n\right)}$$ を求めよ。

（解答）

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{2^m\left(2^m+2^n\right)}$$

$$=\sum_{m=1}^{\infty}\left\{\frac{1}{2^m\left(2^m+2^1\right)}+\frac{1}{2^m\left(2^m+2^2\right)}+\frac{1}{2^m\left(2^m+2^3\right)}+\cdots\right\}$$

$$=\left\{\frac{1}{2^1\left(2^1+2^1\right)}+\frac{1}{2^1\left(2^1+2^2\right)}+\frac{1}{2^1\left(2^1+2^3\right)}+\cdots\right\}$$

$$+\left\{\frac{1}{2^2\left(2^2+2^1\right)}+\frac{1}{2^2\left(2^2+2^2\right)}+\frac{1}{2^2\left(2^2+2^3\right)}+\cdots\right\}$$

$$+\left\{\frac{1}{2^3\left(2^3+2^1\right)}+\frac{1}{2^3\left(2^3+2^2\right)}+\frac{1}{2^3\left(2^3+2^3\right)}+\cdots\right\}$$

$+\cdots$

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{2^m\left(2^m+2^n\right)}$$

$$=\sum_{n=1}^{\infty}\left\{\frac{1}{2^1\left(2^1+2^n\right)}+\frac{1}{2^2\left(2^2+2^n\right)}+\frac{1}{2^3\left(2^3+2^n\right)}+\cdots\right\}$$

$$=\left\{\frac{1}{2^1\left(2^1+2^1\right)}+\frac{1}{2^2\left(2^2+2^1\right)}+\frac{1}{2^3\left(2^3+2^1\right)}+\cdots\right\}$$

$$+\left\{\frac{1}{2^1\left(2^1+2^2\right)}+\frac{1}{2^2\left(2^2+2^2\right)}+\frac{1}{2^3\left(2^3+2^2\right)}+\cdots\right\}$$

$$+\left\{\frac{1}{2^1\left(2^1+2^3\right)}+\frac{1}{2^2\left(2^2+2^3\right)}+\frac{1}{2^3\left(2^3+2^3\right)}+\cdots\right\}$$

$+\cdots$

より

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{2^m(2^m+2^n)}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{2^m(2^m+2^n)}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{2^n(2^n+2^m)}$$

$S=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{2^m(2^m+2^n)}$ とおくと、

$$2S=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{2^m(2^m+2^n)}+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{2^n(2^n+2^m)}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{2^m+2^n}{2^m2^n(2^m+2^n)}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{2^m2^n}$$

$$=\sum_{m=1}^{\infty}\frac{1}{2^m}\sum_{n=1}^{\infty}\frac{1}{2^n}=\frac{\frac{1}{2}}{1-\frac{1}{2}}\cdot\frac{\frac{1}{2}}{1-\frac{1}{2}}=1$$

よって、$S=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{2^m(2^m+2^n)}=\frac{1}{2}$