

$\frac{x^{12}-1}{x-1}$  を整式の範囲で因数分解せよ。

(解答)

$$\begin{aligned}\frac{x^{12}-1}{x-1} &= x^{11} + x^{10} + x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 \\ &= x^{11} + x^{10} + x^9 + x^8 + x^7 + x^6 + (x^5 + x^4 + x^3 + x^2 + x + 1) \\ &= x^6(x^5 + x^4 + x^3 + x^2 + x + 1) + (x^5 + x^4 + x^3 + x^2 + x + 1) \\ &= (x^6 + 1)(x^5 + x^4 + x^3 + x^2 + x + 1) \\ &= (x^2 + 1)(x^4 - x^2 + 1)\{x^3(x^2 + x + 1) + (x^2 + x + 1)\} \\ &= (x^2 + 1)(x^4 - x^2 + 1)(x^3 + 1)(x^2 + x + 1) \\ &= (x^2 + 1)(x^4 - x^2 + 1)(x + 1)(x^2 - x + 1)(x^2 + x + 1) \\ &= (x + 1)(x^2 + 1)(x^2 + x + 1)(x^2 - x + 1)(x^4 - x^2 + 1)\end{aligned}$$

$x^2 \pm x + 1 = \left(x \pm \frac{1}{2}\right)^2 + \frac{3}{4} > 0$  より  $x^2 \pm x + 1 = 0$  は実数解を持たず、

$x^4 - x^2 + 1 = \left(x^2 - \frac{1}{2}\right)^2 + \frac{3}{4} > 0$  より  $x^4 - x^2 + 1 = 0$  は実数解を持たないので、

$$\frac{x^{12}-1}{x-1} = (x+1)(x^2+1)(x^2+x+1)(x^2-x+1)(x^4-x^2+1)$$