nを自然数とし、$f(n)=(x+y)^n-(x^n+y^n)$で定義する。

$f(3), f(5), f(7)$を整式の範囲でそれぞれ因数分解せよ。

（解答）

$$f(3)=(x+y)^3-(x^3+y^3)=3x^2y+3xy^2=3xy(x+y)$$

$$f(5)=(x+y)^5-(x^5+y^5)=5x^4y+10x^3y^2+10x^2y^3+5xy^3$$

$$=5xy(x^3+2x^2y+2xy^2+y^3)=5xy(x^3+y^3+2x^2y+2xy^2)$$

$$=5xy\{(x+y)(x^2-xy+y^2)+2xy(x+y)\}=5xy(x+y)(x^2-xy+y^2)$$

$$f(7)=(x+y)^7-(x^7+y^7)=7x^6y+21x^5y^2+35x^4y^3+35x^3y^4+21x^2y^4+7xy^6$$

$$=7xy(x^5+3x^4y+5x^3y^2+5x^2y^3+3xy^3+y^5)$$

$$=7xy(x^5+y^5+3x^4y+3xy^3+5x^3y^2+5x^2y^3)$$

$$=7xy\{(x+y)(x^4-x^3y+x^2y^2-xy^3+y^4)+3xy(x+y)(x^2-xy+y^2)$$

$$+5x^2y^2(x+y)\}$$

$$=7xy(x+y)\{(x^4-x^3y+x^2y^2-xy^3+y^4)+3xy(x^2-xy+y^2)+5x^2y^2\}$$

$$=7xy(x+y)(x^4+2x^3y+3x^2y^2+2xy^3+y^4)$$

$$=7xy(x+y)(x^4+2x^2y^2+y^4-x^2y^2+2x^3y+2x^2y^2+2xy^3)$$

$$=7xy(x+y)\left\{(x^2+y^2)^2-x^2y^2+2xy(x^2+xy+y^2)\right\}$$

$$=7xy(x+y)\{(x^2+y^2-xy)(x^2+y^2+xy)+2xy(x^2+xy+y^2)\}$$

$$=7xy(x+y)(x^2+xy+y^2)^2$$